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METHOD FOR DETERMINING SIZE AND SHAPE OF OPTICALLY BLACK EMITTING SYSTEMS

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Аннотация—В результате анализа среднеэлементарного и среднелокального коэффициентов температурного излучения дан метод и прибор, ведущие к решению конфигурационных задач в случае лучистого теплообмена.

NOMENCLATURE

F_k, F_i, areas of arbitrary surfaces k and i;
H_k, area of a plane closing surface k where it is concave;
T_k', absolute temperature of the surface k and Δk as small a portion of T_k as the surface Δi is of the surface i;
$T_{\Delta i,y}$, absolute temperature of the element Δi (provided that $T_{k,1} = T_{\Delta i}$);
σ_0, Stefan–Boltzman constant.

VARIOUS methods for an analytical calculation of radiant heat transfer in furnaces, dryers and other apparatuses embodying different shapes and sizes have arisen and are developed on the basis of differential, integral and integro-differential equations.

The inverse problems, however, are of the most practical interest, i.e. determination of size and shape of radiating systems from the distribution of radiation fluxes or temperatures on their boundaries. It is easy to show that the analytical methods mentioned do not lead to a solution of such problems.

Below follows an experimental method allowing not only a statement of, but also the solution of configuration problems.

It is well known [1], that the general case of a three-dimensional problem of an experimental relation for an average angular factor $\varphi_{i,k}$ is of the form:

$$\varphi_{i,k} = \frac{F_k + H_k}{F_i}\left(\frac{T_k}{T_i}\right)^4. \tag{1}$$

Similar to equation (1) introduce a concept of experimental elementary mean, $\varphi_{\Delta i,\Delta k}$, and local mean, $\varphi_{\Delta i,k}$; $\varphi_{\Delta k,i}$, angular factors

$$\varphi_{\Delta i,\Delta k} = \varphi_{\Delta k,\Delta i} = 2\left(\frac{T_k'}{T_i}\right)^4 \tag{2}$$

$$\varphi_{\Delta i,k} = \frac{F_k + H_k}{F_{\Delta i}}\left(\frac{T_k'}{T_i}\right)^4 \tag{3}$$

$$\varphi_{i,\Delta k} = 2\,\frac{F_{\Delta k}}{H_i}\left(\frac{T_k}{T_i}\right)^4. \tag{4}$$

Any complicated radiating system of $i,k = 1, 2, 3 \ldots n$ surfaces may be split up into simple radiating systems each consisting of a couple of finite surfaces i and k. Apparently, an inverse operation is also possible in the case where size and configuration of simple radiating systems are expressed in terms of the conditions of a problem.

Keeping this in mind, consider the simplest radiating systems, different cases of which are presented by solving the following two problems.

The first problem

The size and shape of an optically black emitting surface i are given. Determine the size and shape of an optically black absorption surface k and its position relative to i in which the density of radiant flux incident the surface k equals $E_{\text{inc},k}$ and makes up a known fraction C of the density of (intrinsic) radiation E_{0i} of the surface i. It is easy to see that in this case absolute temperatures of the given and desired surfaces will be expressed as:

$$T_i = \sqrt[4]{(E_{\text{inc}.k}/C\sigma_0)}, \tag{5}$$

$$T_k = \sqrt[4]{(E_{\text{inc}.k}/2\sigma_0)}. \tag{6}$$

All future operation are reduced to finding out co-ordinates of those three-dimensional points relative to the surface i, to which the temperature T_k corresponds. It is clear that the co-ordinates of these points will describe the desired surface k. A special device, co-ordimeter, is illustrated in Fig. 1 (a), and Fig. 1 (b) presents its electric circuit. With the help of this device polar co-ordinates of the mentioned points: two angles α, β and the radius-vector ρ, are easily determined. A model of the given black surface i, being simultaneously an electric heater, is placed in a working quadrant of the co-ordimeter and is heated by an electric current up to the temperature T_i determined by a thermocouple a_1. The part of a radius-vector in the co-ordimeter is played by an expansion rod (ρ) with a fulcrum base (ш) and a thermocouple Δk on the end (a thin metallic circular blackened disk 5 mm in diameter in the centre of which a bead of a thermocouple a_2 is mounted). Transporting the thermoprobe under the surface i into those points of space in which it indicated temperature T_k and observing length of the radius-vector here, ρ, and angles, α and β, which determine its direction, we find the co-ordinates of the points of the desired surface k.

The second problem

Configuration and size of an optically black absorption surface k are given. Determine the size, shape and position relative to k of a black emitting surface i and its temperature, if it is known, that the density of radiant flux, incident on surface k, equals $E_{\text{inc}.k}$, and the largest distance between the surfaces is assumed to be h.

An experimental temperature of the surface k is determined from the following expression

$$F_k E_{\text{inc}.k} = \sigma_0 (F_k + H_k) T_k^4, \tag{7}$$

whence

$$T_k = \sqrt[4]{[E_{\text{inc}.k}/\sigma_0 (1 + H_k/F_k)]}. \tag{8}$$

The following relation gives the temperature $T_{\Delta i}$ of the element $F_{\Delta i}$:

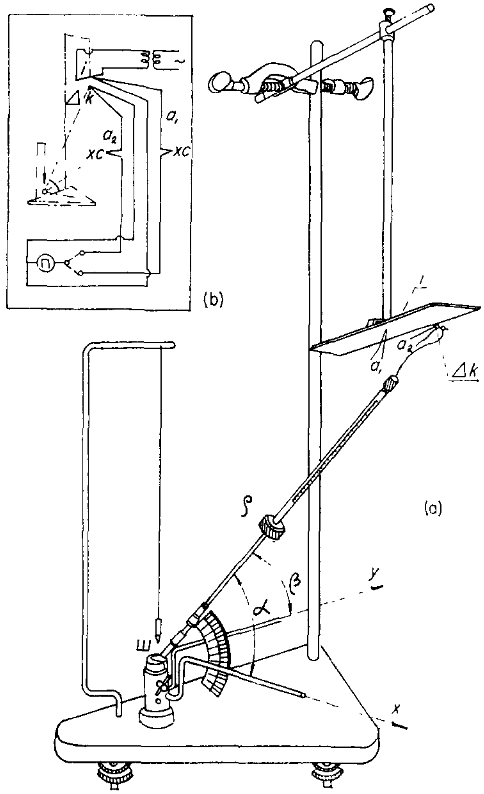

FIG. 1.

$$\varphi_{i,k} = (F_k + H_k)/H_i\,(T_k/T_i)^4 \tag{9}$$

$$\varphi_{\Delta i,k} = (F_k + H_k)/F_{\Delta i}\,(T_k/T_{\Delta i})^4. \tag{10}$$

Since

$$\varphi_{\Delta i,\,k} \approx \varphi_{i,k} \tag{11}$$

then

$$T_{\Delta i} = T_i\,\sqrt[4]{(H_i/F_{\Delta i})}. \tag{12}$$

One would think that knowledge of $T_k = \text{const.}$ and h makes it possible to find $T_{\Delta i} = \text{const.}$ and, consequently, H_i, F_i and T_i.

From formulas (12) and (9) it is seen, however, that the temperature $T_{\Delta i}$ may appear to be very great and practically unattainable. Therefore we take advantage of the following opportunity. Let the given surface k be heated up to $T_{k,1} = T_{\Delta i}$, then local mean angular factor will be expressed as

$$\varphi_{\Delta i,k} = \varphi_{k,\Delta i}\,H_k/F_{\Delta i} = 2\,(T_{\Delta i,y}/T_{\Delta i})^4. \tag{13}$$

On the other hand

$$\varphi_{\Delta i,k} = (H_k + F_k)/F_{\Delta i}\,(T_k/T_{\Delta i})^4. \qquad (14)$$

Comparison between the latter two relations gives:

$$T_{\Delta i,y} = T_k \sqrt[4]{[(H_k + F_k)/2F_{\Delta i}]}. \qquad (15)$$

Preserving the circuit of the previous experiment, put a thermoprobe relative to the surface k at the distance h and heat the surface k till the thermoprobe shows the temperature $T_{\Delta i,y}$. It is evident that the surface k will have the temperature $T_{k,1} = T_{\Delta i} =$ const., which is easily determined from the experiment. Further moving the thermoprobe under the surface k permits determination of co-ordinates of the point in which the thermoprobe shows the temperature $T_{\Delta i,y}$ by the known method. Obviously, the greater the number of points, the more detailed the determination of the size and shape of the desired surface i, i.e. H_i and F_i. The true temperature will be determined from formula (12). As analysis and experiment have shown [1], the most accurate experimental results are obtained when the temperature of a thermoprobe is 323–333°K.

In all cases such conditions of an experiment are easily attainable with the help of the following explicit relation:

$$T_k/T_i = T_k'/T_i' = T_k''/T_i'' = \text{const.} \qquad (16)$$

Now the problem arises how to reproduce the desired surface by co-ordinates α, β, ρ, known from the experiment.

This problem may be solved graphically as in geodesy or cartography [2, 3]. However, being consistent, we show that the solution of this finite problem may be achieved experimentally as well, as follows. A box with solid paraffin is placed in the working quadrant of the co-ordimeter, a thermoprobe is replaced by a chuck with a bit (Fig. 2). Using two co-ordinates, α and β, we give a corresponding direction to the radius-vector and then with the help of a special rotating device enlarge its length up to the length of the third co-ordinate ρ. During this process the bit penetrates into paraffin to some depth. Evidently, in the system of co-ordinates of the device the bottom of a hole will coincide with the desired surface.

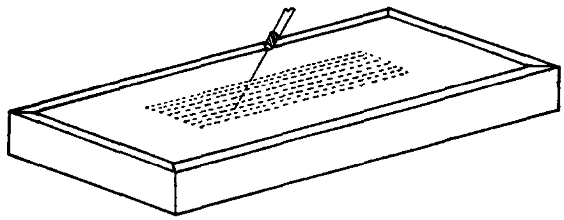

FIG. 2.

Carrying out such an operation for all the points, polar co-ordinates of which are determined, we obtain a system of holes in paraffin; the bottom of each of them is an element of the desired surface, the shape and size of which will be defined by the sum total of these elements, if they are united in a certain way. The latter is attained by removing superfluous paraffin, so that a shape appears in the box.

With the help of gypsum (plaster of Paris) it is easy to replace the paraffin by a stronger metallic shape and die, and from the latter, to arrive at the model of a real surface made of any material, for example, thin brass sheet.

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Abstract—A method is described and apparatus suggested making it possible to solve configuration problems for radiant heat transfer as a result of the analysis of elementary mean and local mean angular factors of temperature radiation.

Résumé—A la suite de l'étude des facteurs angulaires moyens élémentaires et locaux de rayonnement thermique, l'auteur décrit une méthode et suggère un appareillage permettant de résoudre les problèmes de forme dans les échanges de chaleur par rayonnement.

Zusammenfassung—Es wird eine Methode und eine Apparatur beschrieben, die es ermöglichen, Konfigurationsprobleme der Temperaturstrahlung mit Hilfe mittlerer Winkelfaktoren zu lösen.